Enrollment No: ______________________ Exam Seat No: ______________________

C. U. SHAH UNIVERSITY

Winter Examination-2019

Subject Name: Engineering Mathematics – 4

Subject Code: 4TE04EMT2 **Branch: B. Tech (Civil, Electrical)**

Semester : 4 **Date : 01/10/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) hD equal to

(A) $\log(1+\Delta)$ (B) $\log(1-\Delta)$ (C) $\log(1+E)$ (D) $\log(1-E)$

b) $\Delta\nabla$ equal to

(A) $\nabla+\Delta$ (B) $\nabla-\Delta$ (C) $\nabla\Delta$ (D) none of these

c) In application of Simpson's $\frac{1}{3}$ rule, the interval of integration for closer approximation should be

(A) odd and small (B) even and small (C) even and large (D) none of these

d) Putting $n=1$ in the Newton – Cote's quadrature formula following rule is obtained

(A) Simpson's rule (B) Trapezoidal rule (C) Simpson's $\frac{3}{8}$ rule

(D) none of these

e) The Gauss elimination method in which the set of equations are transformed into triangular form.

(A) True (B) False

f) Jacobi iteration method can be used to solve a system of non – linear equations.

(A) True (B) False

g) _______________ is the best for solving initial value problems:

(A) Taylor's series method (B) Euler's method

(C) Runge-Kutta method of 4^{th} order (D) Modified Euler's method

h) The first approximation y_1 of the initial value problem $\frac{dy}{dx}=x^2+y^2$, $y(0)=0$ obtain by Picard's method is

(A) x^2 (B) $\frac{x^2}{2}$ (C) $\frac{x^3}{3}$ (D) none of these

i) The Fourier sine transform of $f(x)=\begin{cases} k, & 0<x<a \\ 0, & x>a \end{cases}$ is

(A) $\sqrt{\frac{2}{\pi}}k\left(\frac{\sin a\lambda}{\lambda}\right)$ (B) $\sqrt{\frac{2}{\pi}}k\left(\frac{1-\cos a\lambda}{\lambda}\right)$ (C) $\sqrt{\frac{2}{\pi}}k\left(\frac{\sin a\lambda}{a}\right)$

(D) none of these

j) The Fourier cosine transform of $f(x)=5e^{-2x}$ is

(A) $\sqrt{\frac{2}{\pi}}\left(\frac{10}{\lambda^2+4}\right)$ (B) $\sqrt{\frac{2}{\pi}}\left(\frac{2}{\lambda^2+4}\right)$ (C) $\sqrt{\frac{2}{\pi}}\left(\frac{10}{\lambda^2-4}\right)$ (D) none of these

k) Which one of the following is an analytic function?

(A) $\mathrm{f}(z)=\mathrm{R}iz$ (B) $\mathrm{f}(z)=\operatorname{Im} z$ (C) $\mathrm{f}(z)=\bar{z}$ (D) $\mathrm{f}(z)=\sin z$

l) Under the transformation $w=\frac{1}{z}$ the image of $|z-2i|=2$ is

(A) $v=\frac{1}{4}$ (B) $v=\frac{-1}{4}$ (C) $|w-2i|=2$ (D) $u^2+v^2=4$

m) If $\vec{V}=(3xyz)i-(2x^2y)j+(2z)k$ then $|\operatorname{div}\vec{V}|$ at $(1,1,1)$ is

(A) 0 (B) 3 (C) 1 (D) 2

n) The tangent vector at the point $t=1$ on the curve $x=t^2+1, y=4t-3, z=t^3$ is

(A) $2i-4j+3k$ (B) $2i+4j+3k$ (C) $2i-4j-3k$ (D) $2i+4j-3k$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Using Newton's divided-difference interpolation, find *f*(1) from the following table: **(5)**

x	– 1	0	2	5	10
y	– 2	– 1	7	124	999

b) Consider following tabular values **(5)**

x	50	100	150	200	250
y	618	724	805	906	1032

Using Newton's Backward difference interpolation formula determine $y(300)$.

c) Find the Fourier sine transform of $f(x)=\begin{cases} 0 & 0<x<a \\ x & a\le x\le b \\ 0 & x>b \end{cases}$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Solve the following system of equations by Gauss-Seidal method. **(5)**

$10x_1+x_2+2x_3=44,\ 2x_1+10x_2+x_3=51,\ x_1+2x_2+10x_3=61$

b) The population of a certain town is shown in the following table: **(5)**

Year	1961	1971	1981	1991	2001
Population (in thousands)	19.96	36.65	58.81	77.21	94.61

Find the rate of growth of population in 1991.

c) Determine the analytic function whose real part is $e^{2x}(x\cos 2y-y\sin 2y)$. **(4)**

Q-4 **Attempt all questions** **(14)**

	a)	Use the fourth – order Runge Kutta method to solve $\frac{dy}{dx} = y - \frac{2x}{y};\quad y(0) = 1$. Evaluate the value of y when $x = 0.2$ and 0.4	**(5)**
	b)	Evaluate $\int_0^{0.6} e^{-x^2}\, dx$ by using Simpson's 1/3rd rule.	**(5)**
	c)	Solve the following system of equations by Gauss-Jordan Method: $5x - 2y + 3z = 18,\ x + 7y - 3z = -22,\ 2x - y + 6z = 22$	**(4)**
Q-5		**Attempt all questions**	**(14)**
	a)	Using Cauchy's integral formula, evaluate $\oint_C \frac{e^{-2z}}{(z+1)^3}\, dz$, where $C : \lvert z \rvert = 2$.	**(5)**
	b)	If $\phi = 45x^2 y$, then evaluate $\iiint_V \phi\, dV$, where V denote the closed region bounded by the planes $4x + 2y + z = 8,\ x = 0,\ y = 0,\ z = 0$.	**(5)**
	c)	Compute $f(9.2)$ by using Lagrange Interpolation formula from the following data:	**(4)**

x	9	9.5	11
y	2.1972	2.2513	2.3979

Q-6		**Attempt all questions**	**(14)**
	a)	Prove that $\vec{F} = (y\cos z - \sin x)i + (x\sin z + 2yz)\, j + (xy\cos z + y^2)k$ is irrotational and find its scalar potential.	**(5)**
	b)	Show that the transformation $w = \frac{1}{z}$ transforms all circles and straight lines into the circles and straight lines in the w-plane, which circles in the z-plane become straight lines in the w-plane, and which straight lines are transformed into other straight lines?	**(5)**
	c)	Using Taylor's series method, compute $y(-0.1)$, $y(0.1)$, $y(0.2)$ correct to four decimal places, given that $\frac{dy}{dx} = y - \frac{2x}{y},\ y(0) = 1$	**(4)**
Q-7		**Attempt all questions**	**(14)**
	a)	Show that the function defined by the equation $f(z) = \begin{cases} u(x,y) + iv(x,y), & \text{if } z \neq 0 \\ 0, & \text{if } z = 0 \end{cases}$ where $u(x,y) = \frac{x^3 - y^3}{x^2 + y^2}$ and $v(x,y) = \frac{x^3 + y^3}{x^2 + y^2}$ is not analytic at $z = 0$ although Cauchy – Riemann equations are satisfied at that poiut.	**(5)**
	b)	If $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^3\hat{k}$, show that $\int_C \vec{F} \cdot d\vec{r}$ is independent of the path of integration. Hence evaluate the integral when C is any path joining A(1, – 2, 1) to B(3, 1, 4).	**(5)**
	c)	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's 3/8 Rule using $h = \frac{1}{6}$.	**(4)**
Q-8		**Attempt all questions**	**(14)**
	a)	Use Euler's method to find an approximate value of y at $x = 0.1$, in five steps,	**(5)**

given that $\frac{dy}{dx} = x - y^2$ and $y(0) = 1$.

b) Find the Fourier cosine and sine integral of $f(x) = e^{-kx}$ $(x > 0,\ k > 0)$. **(5)**

c) Find the angle between the tangents to the curve $x = t^2, y = 2t, z = -t^3$ at the points $t = 1$ and $t = -1$. **(4)**